advanced network
science initiative
(ansi)

Beyond pairwise Ising models in D-Wave: searching for hidden multi-body interactions

Yaroslav Kharkov[1,2], Andrey Lokhov[1,3] (PI), Marc Vuffray[3]

with Misha Chertkov[1,3], Carleton Coffrin[4], Sidhant Misra[5]

[1] CNLS [2] UNSW Sydney [3] T-4 [4] A-1 [5] T-5

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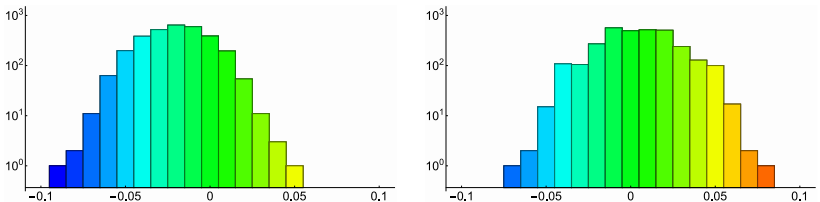
Introduction and context: de-biasing the machine

Project on the D-Wave calibration: emerged from the previous ISTI call in 2016

Observation: input couplings $\vec{J} = \{J_{ij}, h_i\}$ are different from the ones realized in D-Wave, due to an "effective temperature" and biases

$$D(\vec{J}) = \vec{J}' = \beta_J(\vec{J} + \delta\vec{J})$$



Goal: create a calibration procedure/software that takes the target couplings $\vec{J}'_{\text{target}}$ and outputs the input couplings to D-Wave $\vec{J}'_{\text{input}}$ [*parallel effort on a different funding*].

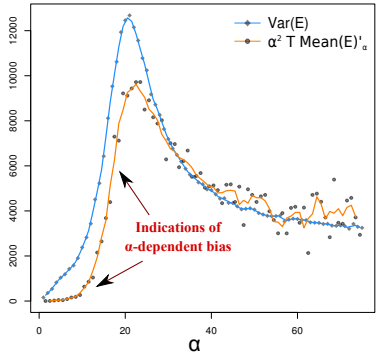
Introduction and context: what is the right model?

Is **statistics of qubits** well described by a classical Ising model? [Important for sampling]

Signature of **biases**: if $P(\underline{\sigma}) \propto e^{-\mathcal{H}(\underline{\sigma})/(\alpha T)}$, then $\alpha^2 T \frac{\partial}{\partial \alpha} \langle \mathcal{H} \rangle = \langle \mathcal{H}^2 \rangle - \langle \mathcal{H} \rangle^2$

Might need to correct for biases in higher-order interaction terms

Checkerboard pattern without magnetic fields

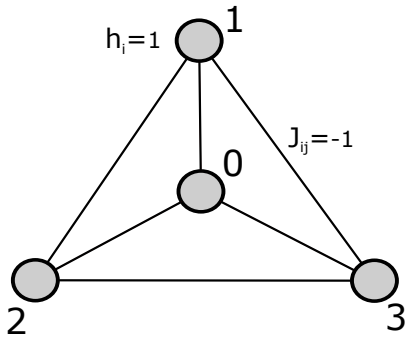


Goals of this project: familiarization with D-Wave, and detecting potential presence of multi-body interactions

Multi-body interactions can exist even when the original model is Ising

Ising model on 4 qubits

$\mathcal{H} = \sum_{i<j} J_{ij}\sigma_i\sigma_j + \sum_i h_i\sigma_i$

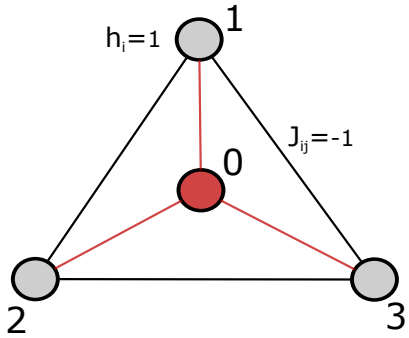


$J_{01}\sigma_0\sigma_1 + J_{02}\sigma_0\sigma_2 + J_{03}\sigma_0\sigma_3 + J_{12}\sigma_1\sigma_2 + J_{13}\sigma_1\sigma_3 + J_{23}\sigma_2\sigma_3 + h_0\sigma_0 + h_1\sigma_1 + h_2\sigma_2 + h_3\sigma_3$

Multi-body interactions can exist even when the original model is Ising

Ising model with a hidden spin

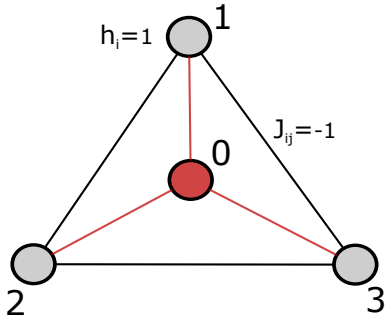
$\mathcal{H} = \sum_{i<j} J_{ij}\sigma_i\sigma_j + \sum_i h_i\sigma_i$



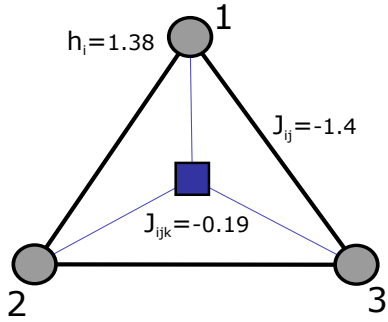
$\sigma_0(J_{01}\sigma_1 + J_{02}\sigma_2 + J_{03}\sigma_3 + h_0) + J_{12}\sigma_1\sigma_2 + J_{13}\sigma_1\sigma_3 + J_{23}\sigma_2\sigma_3 + h_1\sigma_1 + h_2\sigma_2 + h_3\sigma_3$

Multi-body interactions can exist even when the original model is Ising

How does the model on 3 observed qubits look like?



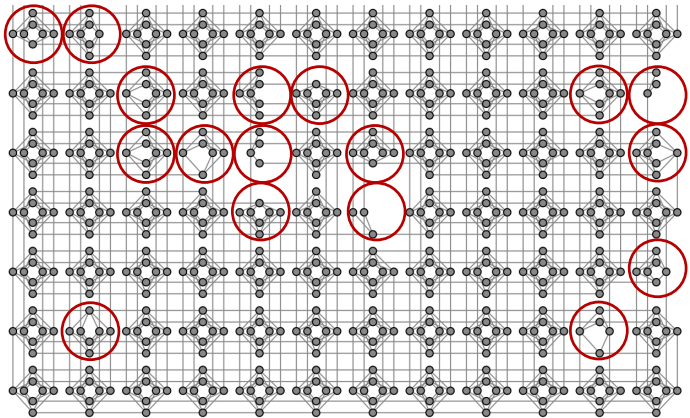
Reconstructed model



$$J_{123}\sigma_1\sigma_2\sigma_3 + J_{12}\sigma_1\sigma_2 + J_{13}\sigma_1\sigma_3 + J_{23}\sigma_2\sigma_3 + h_1\sigma_1 + h_2\sigma_2 + h_3\sigma_3$$

Why do we care?

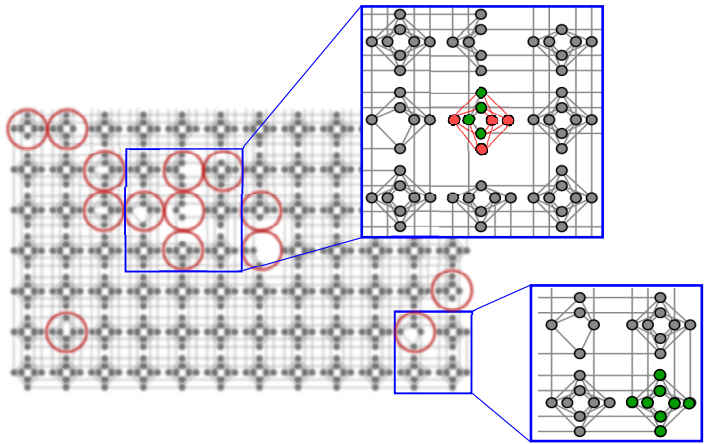
Let us take a close look at the D-Wave chip:



Even if the underlying model is of the pairwise Ising type, multi-body interactions can still be present due to broken qubits

Detection of multi-body interactions: experimental setup

Study of two cell types: with and without broken qubits

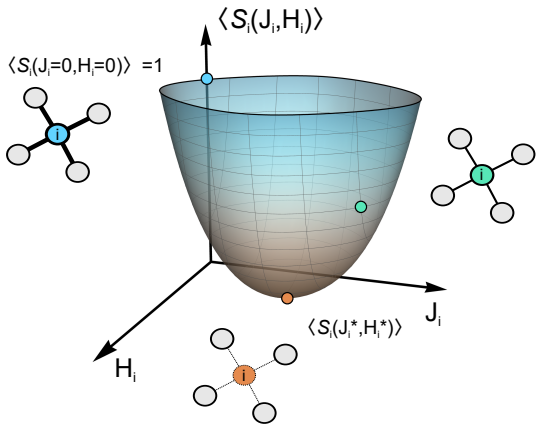


And how do we do that? Interaction Screening method

For each spin, minimize the potential $S_i(J_i, H_i)$ which applies counter-interactions ($P \propto e^{-\mathcal{H}}$):

$$(\widehat{J}_i, \widehat{H}_i) = \underset{(J_i,H_i)}{\operatorname{argmin}}\, S_i(J_i, H_i)$$

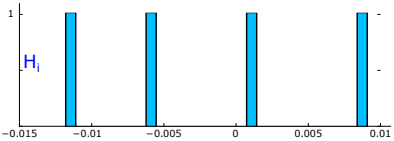
$$S_i(J_i, H_i) = \langle \exp(\sum_{j,k\neq i} J_{ijk}\sigma_i\sigma_j\sigma_k + \sum_{j\neq i} J_{ij}\sigma_i\sigma_j + H_i\sigma_i) \rangle_M$$

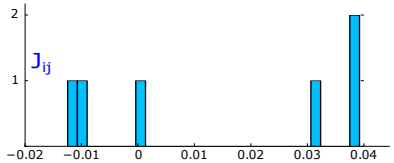


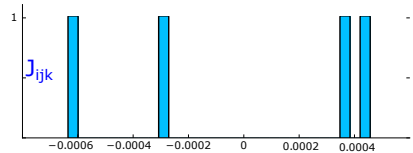
Vuffray, Misra, Lokhov, Chertkov, NIPS (2016)
Lokhov, Vuffray, Misra, Chertkov, arXiv:1612.05024 (2016)

Results: zero input Hamiltonian

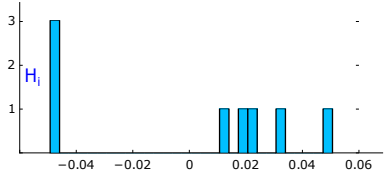
Cell with broken qubits:

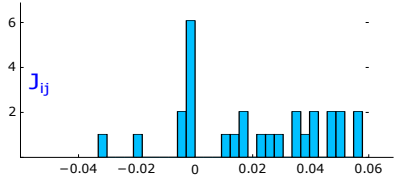


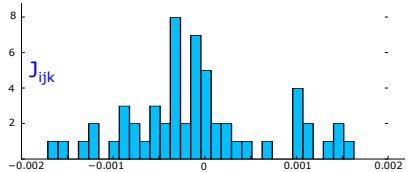




Cell w/o broken qubits:







Some facts and remaining questions

✓ 4-body interactions and beyond are indistinguishable from zero

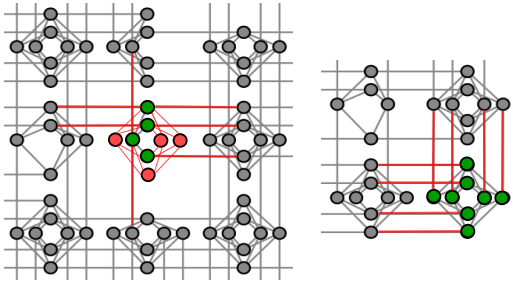
✓ The amplitude of detected 3-body terms J_{ijk} is at least order of magnitude smaller than residuals J_{ij}, H_i, and does not seem to change for different input Hamiltonians

✓ **Conclusion:** higher-order correction terms can be ignored - good news

✓ **Question:** Why the values of J_{ijk} detected in cells with and without broken spins are of the same order? After all, are these 3-body interaction induced or intrinsic?

"Screening" cells from external influence: experimental design

Induced or **intrinsic** nature of the detected 3-body interactions?



Possibility: neighboring nodes still carry out residual couplings, and need to be "screened" using the pairwise calibration procedure

Future work: this question of academic interest is left for future work.